A New Surrogate Assisted Multi-Objective Optimization Algorithm for an Axial Flux Permanent Magnet Synchronous Motor Design

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In this paper, a new surrogate assisted multi-objective optimization (SAMOO) algorithm is presented to optimize an axial flux permanent magnet synchronous motor (AFPMSM) design. The proposed algorithm is a multi-objective algorithm (MOO) that can both maximize the torque amplitude and minimize the torque ripple to improve the power transmission efficiency and stability of an AFPMSM control considering various design variables. While the conventional MOO algorithms have a serious problem of requiring too many function calls especially considering many design variables, the proposed algorithm can make an accurate and a well-distributed Pareto front set with fewer function evaluations. The superiority of the proposed algorithm is verified by comparing with conventional MOO algorithms. Finally, the proposed algorithm is applied to an optimal design process of an AFPMSM.

Index Terms— **Axial flux permanent magnet synchronous motor, kriging, multi-objective, surrogate model.**

I. Introduction

To OPTIMAL design for the electric machines, various aspects such as efficiency, power, and cost should be considered. Hence, many researchers have studied multiple-objectives optimization (MOO) in which the goal is to minimize or maximize several conflicting objective functions simultaneously [1]. Especially, nondominated sorting genetic algorithm II (NSGA-II) and multi-objective particle swarm optimization (MOPSO) have been popularly used [1], [2].

However, conventional algorithms require many function calls to solve MOO problems, which dramatically increase the optimization time due to the use of a finite element method (FEM) at every function call. Especially in an electric machine with many design variables, the optimization time proportionally increases since FEM analysis is required at every function call [3], [4].

To address this problem, we propose a new surrogate assisted MOO (SAMOO) algorithm. By using kriging surrogate model and solution searching based on curve fitting method, the computational time can be significantly reduced even considering many design variables [3]-[5]. The proposed algorithm concentrates on improving the convergence speed, the well-distributed Pareto front set, and solution diversity. Its superior performance is verified by comparing with conventional MOO algorithms with a test function.

Furthermore, the proposed algorithm is applied to the design process of an axial flux permanent magnet synchronous motor (AFPMSM).

II. Proposed Algorithm

A. Kriging surrogate model

In the kriging method, the estimated value, z^*, is a weighted linear combination of given n sample points as given by

$$z^* = \sum_{i=1}^{n} \lambda_i z_i \tag{1}$$

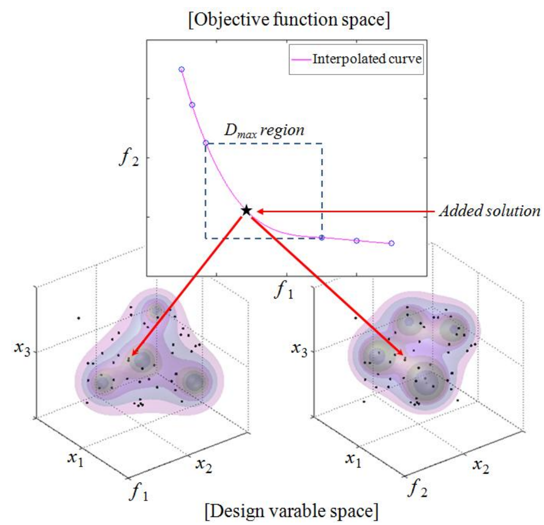

Fig. 1. Solution searching by using curve fitting.

where z_i and λ_i are the i^{th} sample point and weight, respectively. The estimation variance of error is defined by

$$\sigma^2 = E\left[\left(z^* - z_o\right)^2\right] = E\left[\left(\sum_{i=1}^{n} \lambda_i z_i - z_o\right)^2\right] \tag{2}$$

where the true value at a location x_o is z_o. Equation (2) is developed, and a constraint is added in order to ensure the unbiased estimation as follows:

$$L = Cov_{oo} - 2\sum_{i=1}^{n} \lambda_i Cov_{io} + \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j Cov_{ij} + 2\omega\left(1 - \sum_{i=1}^{n} \lambda_i\right) \tag{3}$$

where ω is a Lagrange multiplier to consider the constraint. When equation (3) is minimized with respect to λ_i and ω, the ordinary kriging equation is obtained. The covariance matrix Cov_{ij} can be defined as

$$Cov_{ij} = \sigma^2 \cdot R\left(\theta, x_i, x_j\right) \tag{5}$$

where R is the correlation matrix. In this research, one of the correlation functions, a Gaussian correlation function which is most commonly used is adopted as follows:

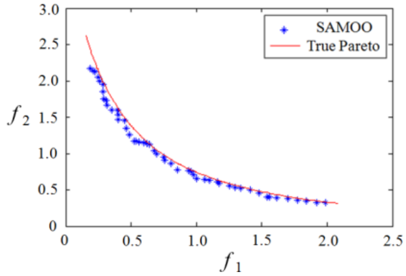

Fig. 2. Pareto front produced by SAMOO for the test function.

$$R(x_i, x_j) = \exp\left[-\sum_{d=1}^{m} \theta_d \left(x_{id} - x_{jd}\right)^2\right] \tag{4}$$

where m is the dimension of design variables and θ is an unknown covariance parameter vector, which influences the effect of covariance function. In order to estimate the best θ, maximum likelihood estimation is used [5].

B. Solution searching based on curve fitting method

The MOO algorithm applied in the FEM analysis should solve the MOO problem with fewer function calls to reduce the computational time. To do this, new solutions should be effectively added in the blank of the Pareto front set [1].

As a blank, D_{max} *region* is defined as the region with maximum distance between the nondominated solutions. The D_{max} is computed by

$$D_{max} = \max_{1 \le j < N_p - 1} \left(\sum_{i=1}^{N_{ob}} \left| \frac{f_{ij} - f_{ij+1}}{max(f_i) - min(f_i)} \right| \right) \tag{5}$$

where f_i is the i^{th} objective function, x_1, x_2, and x_3 represent the design variables, N_p is the number of nondominated solutions, and N_{ob} is the number of objective functions. The dotted region in Fig. 1 represents the D_{max} *region*.

The cubic spline interpolation model constructed by the nondominted solutions is used for the effective addition of solution in the blank of the Pareto front set. The pink line in Fig. 1 shows the interpolated curve reflecting the Pareto front set in the objective function space. In Fig. 1, *Added solution* is located on the interpolated curve in the D_{max} *region*.

The design variables are inversely searched by comparing the objective values of the surrogate model to those of *Added solution*, the minimum design variables difference, ε_{min}, is searched by

$$\varepsilon_{min} = \min_{i,j,k} \sum_{l=1}^{N_{ob}} \left| f_{l,add} - f_l(x_1(i), x_2(j), x_3(k)) \right| \tag{6}$$

where $i \in \{1, 2, 3, \cdots, N_{gridx1}\}$, $j \in \{1, 2, 3, \cdots, N_{gridx2}\}$, and $k \in \{1, 2, 3, \cdots, N_{gridx3}\}$. N_{gridx1}, N_{gridx2}, and N_{gridx3} are the number of x_1, x_2, and x_3 grids, respectively and $f_{l,add}$ is the l^{th} objective value of the *added solution*.

Due to the direct addition of the solution in the D_{max} *region*, the well-distributed Pareto front set can be made with fewer function evaluations even considering many design variables.

III. Numerical Test and Result

To verify the proposed algorithm, conventional algorithms such as NSGA-II and MOPSO and the proposed SAMOO are applied into a test function as follows:

Maximize $F = (f_1(x_1, x_2, x_3), f_2(x_1, x_2, x_3))$,

$$f_1 = \frac{(x_1 x_2 x_3)}{(3x_1 + 4x_2 + 5x_3)}, \quad f_2 = \frac{(x_1 + 4x_3)}{(3x_1 x_2 + x_3)} \tag{7}$$

where $1 < x_1, x_2, x_3 < 5$. Fig. 2 shows the result by the proposed algorithm by applying five hundred function evaluations in the test function. It can be seen that SAMOO can realize the well-distributed Pareto front set that encompasses the entire the Pareto front set. Additionally, the proposed algorithm is the best with respect to *GD*, *SP*, and requires less number of function calls as shown in TABLE I. *GD* denotes the distance between the constructed Pareto front set by using algorithms and the real Pareto front set. *SP* represents standard deviation of distance among solutions in the Pareto front set [2].

TABLE I
Comparison Average Results for One Hundred Times Test

	GD	*SP*	No. function calls
NSGA-II	0.0083	0.0432	1055
MOPSO	0.0097	0.0551	940
SAMOO	0.0056	0.0243	502

IV. Optimal IPMSM Design by SAMOO

In this section, SAMOO is applied in the AFPMSM design to validate the feasibility. The detail explanations about the application will be in the full paper.

V. Conclusion

Most of the electric machine design is the MOO problem in which various conflict objectives should be taken into account simultaneously. However, widely used conventional MOO algorithms require more function evaluations.

Hence, this research has significant meaning due to the fact that the proposed algorithm not only considerably reduces the number of function calls, but also realizes the well-distributed Pareto front set considering many design variables. Furthermore, the feasibility of the optimal design using the proposed algorithm is verified by the AFPMSM optimal design.

For these reasons, this research is noteworthy in that the proposed algorithm can be widely used in a variety of electric machine designs using FEM.

References

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II”, *IEEE Trans. Evol.*, vol. 6, no. 2, pp. 182-197, Apr. 2002.
[2] C. A. C. Coello, G. T. Pulido, and M. S. Lechuga, “Handling multiple objectives with particle swarm optimization”, *IEEE Trans. Evol.*, vol. 8, no. 3, pp. 256-278, Jun. 2004.
[3] D. K. Lim, D. K. Woo, I. W. Kim, J. S. Ro, and H. K. Jung, “Cogging torque minimization of a dual-type axial-flux permanent magnet motor using a novel optimization algorithm”, *IEEE Trans. Magnetics*, vol. 49, no. 9, pp. 5106-5111, Sep. 2013.
[4] D. K. Woo, J. H. Choi, M. Ali, and H. K. Jung, “A novel multimodal optimization algorithm applied to electromagnetic optimization”, *IEEE Trans. Magnetics*, vol. 47, no. 6, pp. 1667-1673, Jun. 2011.
[5] B. Xia, M. T. Pham, Y. Zhang, and C. S. Koh, “A global optimization algorithm for electromagnetic devices by combining adaptive taylor kriging and particle swarm optimization”, *IEEE Trans. Magnetics*, vol. 49, no. 5, pp. 2061-2064, May 2013.